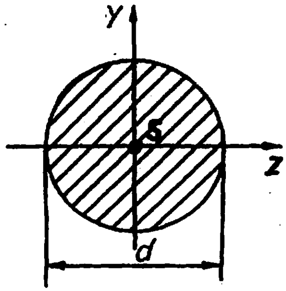
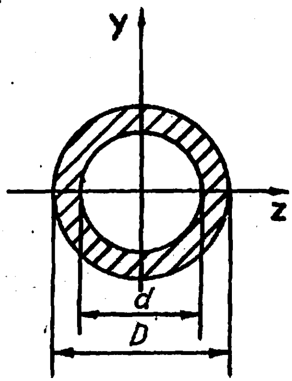
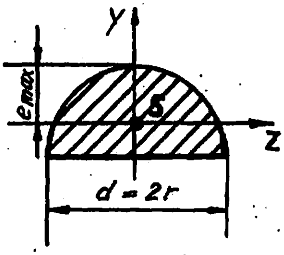
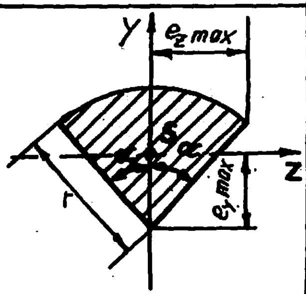
<table>
<tr>
<td>Institut für Materialwissenschaft und Werkstofftechnologie
Professur für Angewandte Mechanik</td>
<td>Statik und Festigkeitslehre
Axiale (äquatoriale) Trägheits- und Widerstandsmomente homogener Flächen - 2</td>
</tr>
</table>

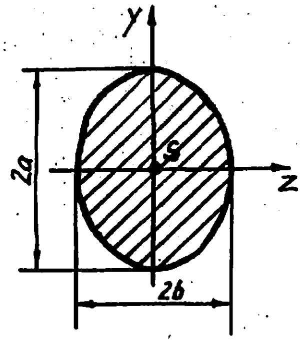
<table>
<tr>
<td>


</td>
<td>$J_z = J_y = \frac{\pi}{64} d^4$
$\approx \frac{1}{20} d^4$</td>
<td>$W_z = W_y = \frac{\pi}{32} d^3$
$\approx \frac{1}{10} d^3$</td>
</tr>
<tr>
<td>


</td>
<td>$J_z = J_y = \frac{\pi}{64} (D^4 - d^4)$
Höhlungsverhältnis: $\alpha = \frac{d}{D}$
Erfahrungswert: $0{,}4 \le \alpha \le 0{,}6$
$J_z = J_y = \frac{\pi}{64} (1 - \alpha^4) D^4$</td>
<td>$W_z = W_y = \frac{\pi}{32} \cdot \frac{D^4 - d^4}{D}$
$W_z = W_y = \frac{\pi D^3}{32} (1 - \alpha^4)$</td>
</tr>
<tr>
<td>


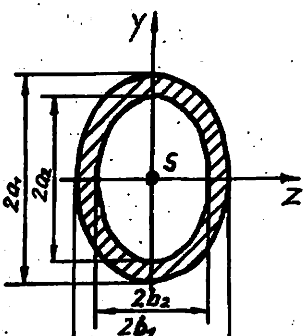
</td>
<td>$J_z = 0{,}1098\, r^4$
$J_y = \frac{\pi}{8} r^4$</td>
<td>$W_z = 0{,}1908 \cdot r^3$
$W_y = \frac{\pi}{8} r^3$
$e_{max} = 0{,}5756\, r$</td>
</tr>
<tr>
<td>


</td>
<td>$J_z = \frac{r^4}{4} \left(\alpha + \sin\alpha \cos\alpha - \frac{16}{9} \frac{\sin^2\alpha}{\alpha} \right)$
$J_y = \frac{r^4}{4} (\alpha - \sin\alpha \cdot \cos\alpha)$</td>
<td>$W_z = \frac{J_z}{e_{y\,max}}$
$W_y = \frac{J_y}{e_{z\,max}}$
$e_{y\,max} = \frac{2}{3} r \frac{\sin\alpha}{\alpha}$
$e_{z\,max} = r \cdot \sin\alpha$</td>
</tr>
</table>



$$J_z = \frac{\pi \cdot a^3 b}{4} \qquad W_z = \frac{\pi \cdot a^2 b}{4}$$

$$J_y = \frac{\pi \cdot a \cdot b^3}{4} \qquad W_y = \frac{\pi a b^2}{4}$$

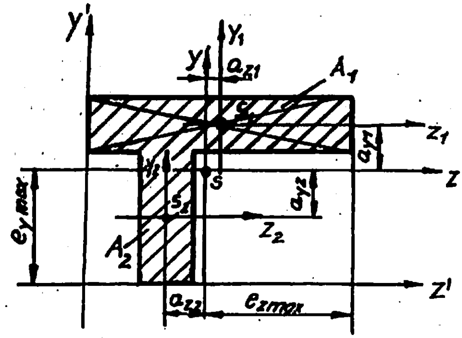


$$J_z = \frac{\pi}{4}(a_1^3 \cdot b_1 - a_2^3 b_2) \qquad W_z = \frac{J_z}{a_1}$$

$$J_y = \frac{\pi}{4}(b_1^3 \cdot a_1 - b_2^3 a_2) \qquad W_y = \frac{J_y}{b_1}$$

weitere Flächen: siehe Fachliteratur

Satz von Steiner



$$J_z = \sum_{i=1}^{n} (J_{zi} + A_i a_{yi}^2)$$

$$J_y = \sum_{i=1}^{n} (J_{yi} + A_i \cdot a_{zi}^2)$$

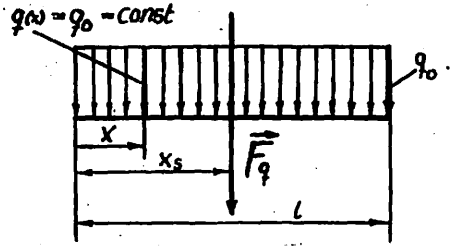
$$W_z = \frac{J_z}{e_{y\,max}}$$

$$W_y = \frac{J_y}{e_{z\,max}}$$

Merke: Der Satz von Steiner darf nur für axiale (äquatoriale) Flächenträgheitsmomente u. nicht für axiale (äquatoriale) Flächenwiderstandsmomente angewendet werden.

Institut für Materialwissenschaft und Werkstofftechnologie Professur für Angewandte Mechanik	Statik und Festigkeitslehre Linienkräfte – 1

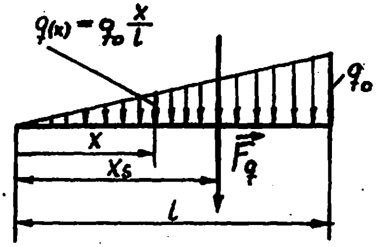
1.



$F_q = q_0 \cdot l$

$x_S = \frac{1}{2} \cdot l$

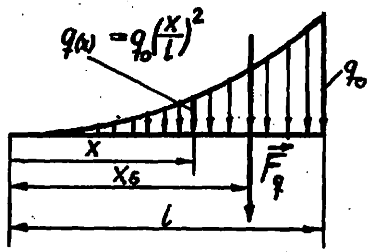
2.



$F_q = \frac{1}{2} q_0 \cdot l$

$x_S = \frac{2}{3} \cdot l$

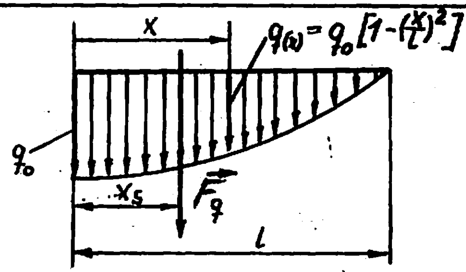
3.



$F_q = \frac{1}{3} q_0 \cdot l$

$x_S = \frac{3}{4} \cdot l$

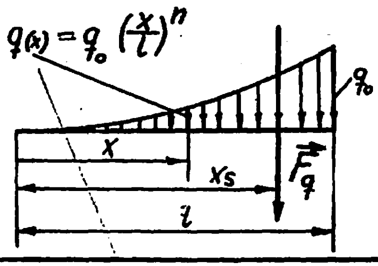
4.



$F_q = \frac{2}{3} q_0 \cdot l$

$x_S = \frac{3}{8} \cdot l$

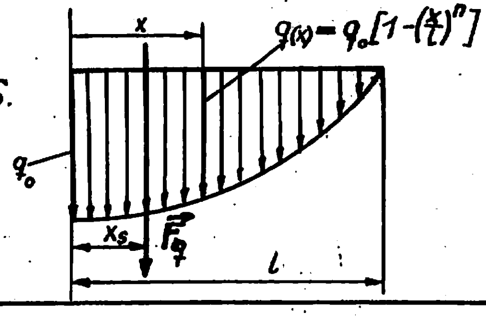
5.



$F_q = \frac{1}{n+1} q_0 \cdot l$

$x_S = \frac{(n+1)}{(n+2)} \cdot l$

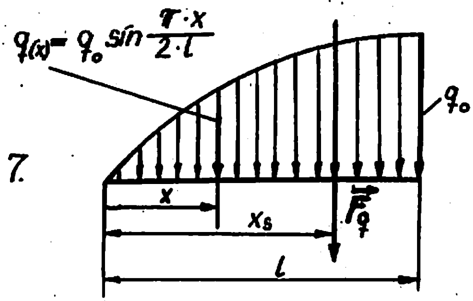
6.



$$F_q = \frac{n}{n+1}\cdot q_0\cdot l$$

$$x_S = \frac{n+1}{2(n+2)}\cdot l$$

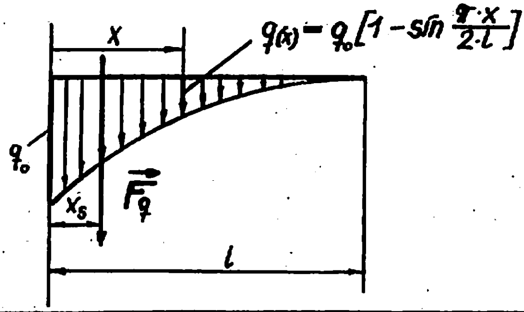
7.



$$F_q = \frac{2}{\pi}\, q_0\cdot l$$

$$x_S = \frac{2}{\pi}\cdot l$$

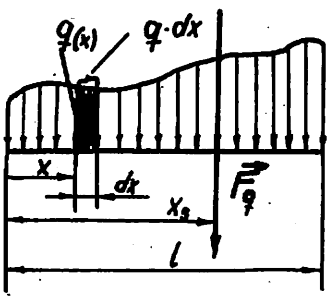
8.



$$F_q = \frac{\pi-2}{\pi}\cdot q_0\cdot l$$

$$x_S = \frac{\pi^2-8}{2\pi(\pi-2)}\cdot l$$

9.

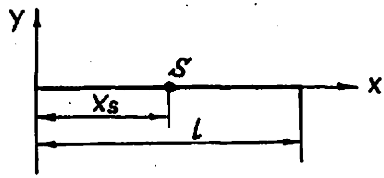


$$F_q = \int_{x=0}^{x=l} q_{(x)}\, dx$$

$$x_S = \frac{1}{F_q}\int_{x=0}^{x=l} x\cdot q_{(x)}\cdot dx$$

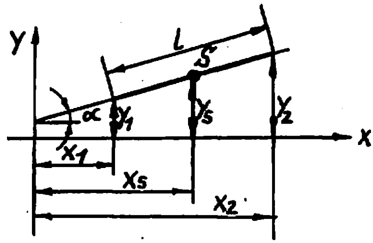
Institut für Materialwissenschaft und Werkstofftechnologie **Professur für Angewandte Mechanik**	**Statik und Festigkeitslehre** **Schwerpunktlagen spezieller ebener Linien**

1.



$$x_s = \frac{1}{2} l$$

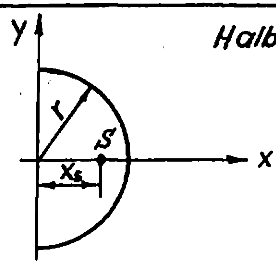
2.



$$x_s = \frac{1}{2}(x_1 + x_2) = x_1 + \frac{1}{2} l \cos\alpha$$

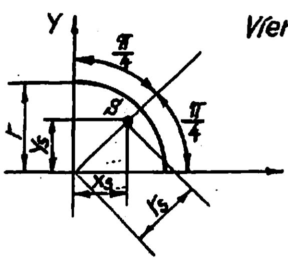
$$y_s = \frac{1}{2}(y_1 + y_2) = y_1 + \frac{1}{2} l \sin\alpha$$

3. *Halbkreis*



$$x_s = \frac{2 \cdot r}{\pi} \quad ; \quad y_s = 0$$

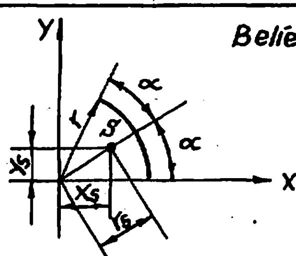
4. *Viertelkreis*



$$x_s = \frac{2 \cdot r}{\pi} \quad ; \quad y_s = \frac{2 \cdot r}{\pi}$$

$$r_s = \frac{2 \cdot r}{\pi} \sqrt{2} = 0{,}9003 \cdot r$$

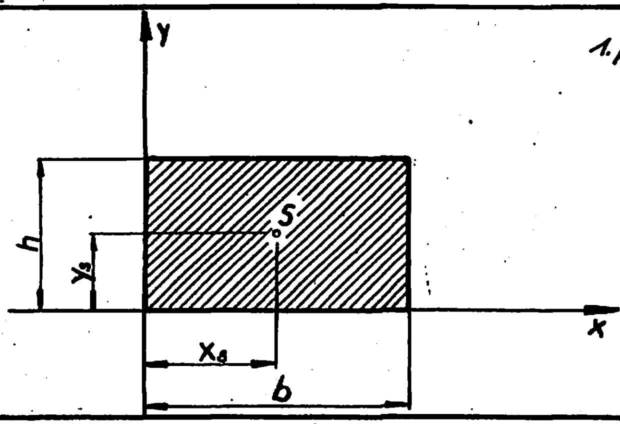
5. *Beliebiges Kreisbogenstück*



$$x_s = r \frac{\sin 2\alpha}{2\hat{\alpha}} \quad ; \quad y_s = r \frac{1 - \cos 2\alpha}{2\hat{\alpha}}$$

$$r_s = r \frac{\sin\alpha}{\hat{\alpha}} \quad ; \quad \hat{\alpha} = \frac{\alpha^\circ}{180} \pi$$

Institut für Materialwissenschaft und Werkstofftechnologie Professur für Angewandte Mechanik	Statik und Festigkeitslehre Schwerpunktlagen spezieller ebener Flächen - 1



1.) Rechteck

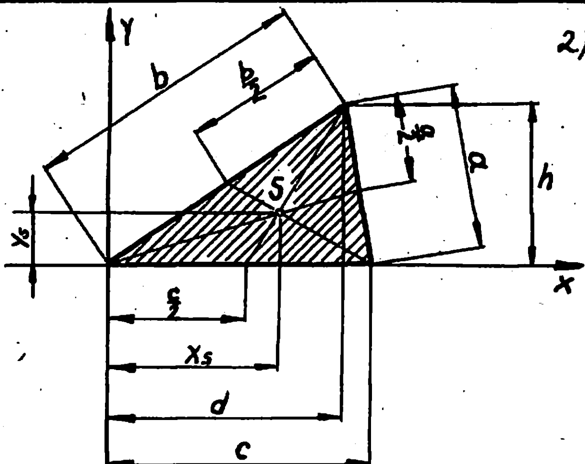
$A = b \cdot h$

$x_s = \frac{b}{2}$

$y_s = \frac{h}{2}$

Quadrat

$b = h = a$

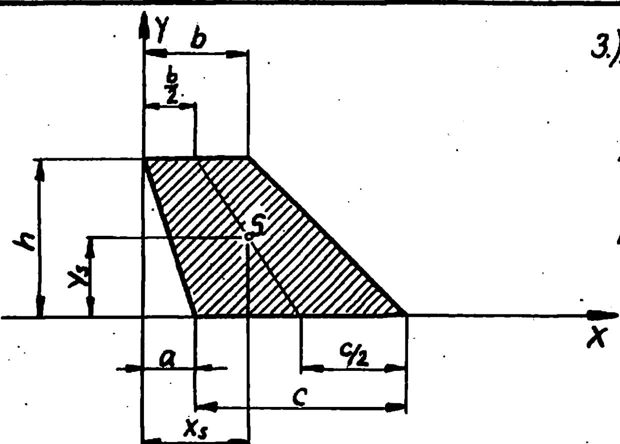


2.) Dreieck

$A = \frac{c \cdot h}{2}$

$x_s = \frac{c+d}{3}$

$y_s = \frac{h}{3}$

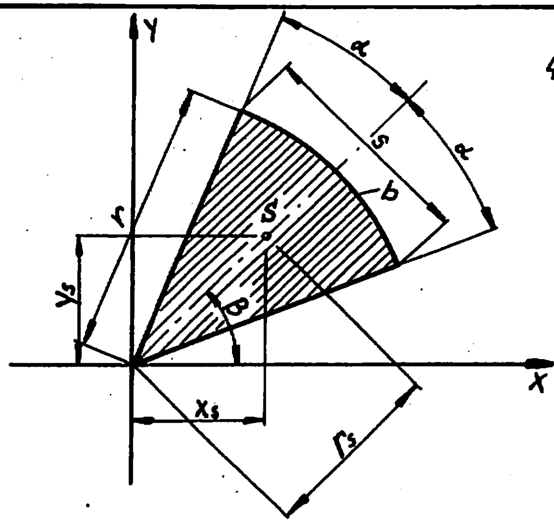


3.) Trapez

$A = \frac{h}{2}(b+c)$

$x_s = \frac{c(c+2a)+b(a+b+c)}{3(b+c)}$

$y_s = \frac{h}{3} \cdot \frac{c+2b}{b+c}$



4.) Kreisausschnitt

$A = r^2 \hat{\alpha}$

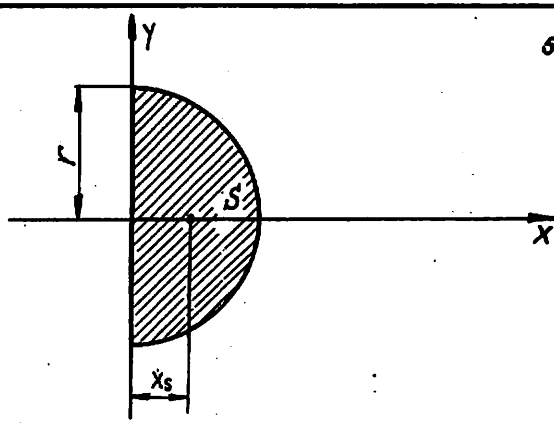
$s = 2 \cdot r \cdot \sin\alpha$

$b = r \cdot 2\hat{\alpha}$

$r_s = \frac{2}{3} r \frac{s}{b} = \frac{1}{3} \frac{r^2 s}{A}$

$x_s = r_s \cos\beta$

$y_s = r_s \sin\beta$

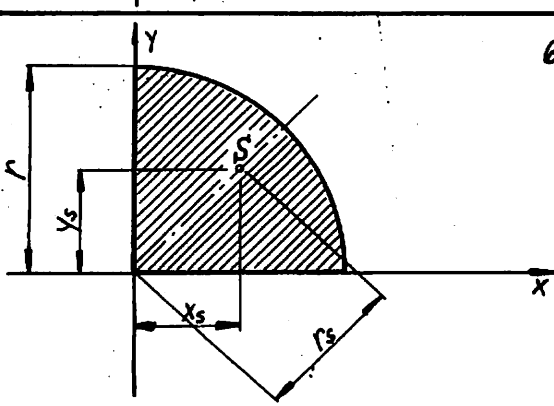


5.) Halbkreis

$A = \frac{1}{2} r^2 \pi$

$x_s = r_s = \frac{4}{3} \frac{r}{\pi}$

$y_s = 0$

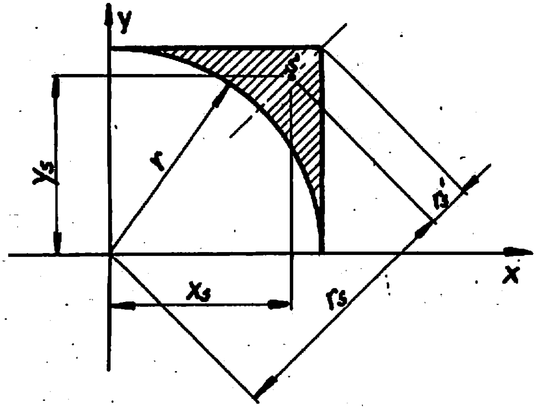


6.) Viertelkreis

$A = \frac{1}{4} r^2 \pi$

$r_s = \frac{4r}{2\pi} \sqrt{2}$

$x_s = y_s = \frac{4r}{3\pi}$



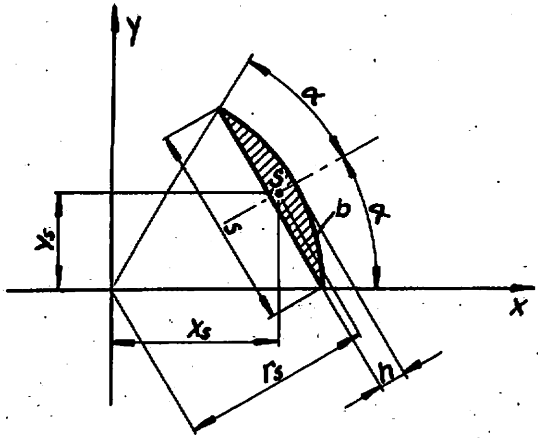
7.) Äußerer Teil des Viertelkreises

$$A = \frac{1}{4} r^2 (4-\pi)$$

$$r_s = \frac{2\sqrt{2}}{3(4-\pi)}\, r$$

$$r_s' = r\sqrt{2} - r_s$$

$$x_s = y_s = \frac{2r}{3(4-\pi)}$$



8.) Kreisabschnitt

$$A = \frac{1}{2} r^2 (2\hat{\alpha} - \sin 2\alpha)$$

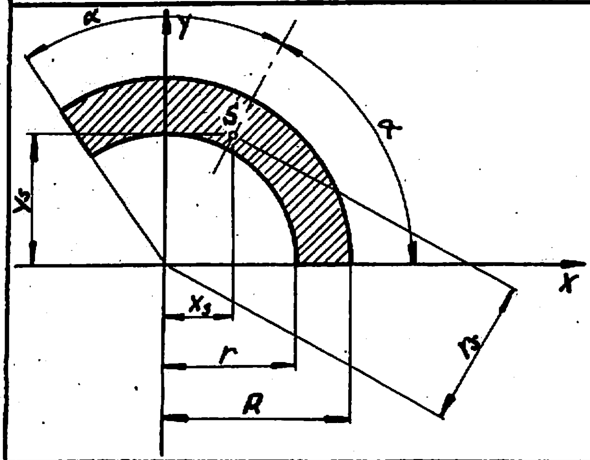
$$r_s = \frac{4}{3} r \frac{\sin^3\alpha}{2\hat{\alpha} - \sin 2\alpha} = \frac{s^3}{12A}$$

$$s = 2r \sin\alpha$$

$$x_s = r_s \cos\alpha$$

$$y_s = r_s \sin\alpha$$

$$h = r(1-\cos\alpha)$$



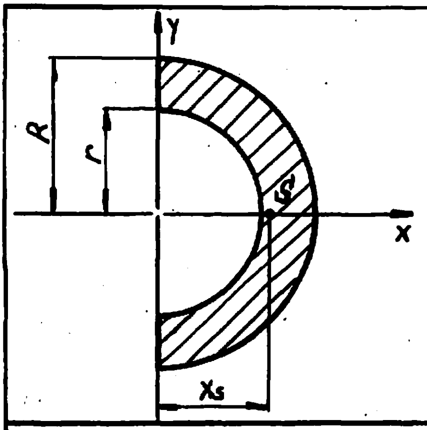
9.) Kreisringstück

$$A = \hat{\alpha}\,(R^2 - r^2)$$

$$r_s = \frac{2}{3} \frac{R^3 - r^3}{R^2 - r^2} \cdot \frac{\sin\alpha}{\hat{\alpha}}$$

$$x_s = r_s \cos\alpha$$

$$y_s = r_s \sin\alpha$$

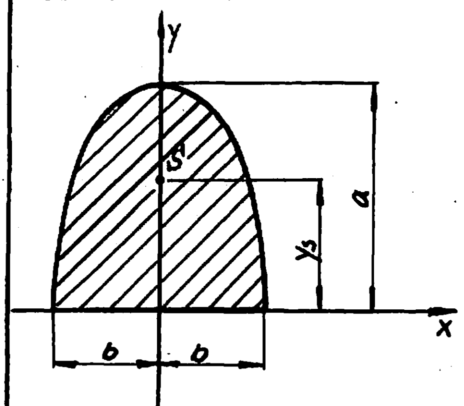


10.) Halber Kreisring

$$A = \frac{1}{2}\pi(R^2 - r^2)$$

$$x_s = \frac{4}{3\pi} \cdot \frac{R^3 - r^3}{R^2 - r^2}$$

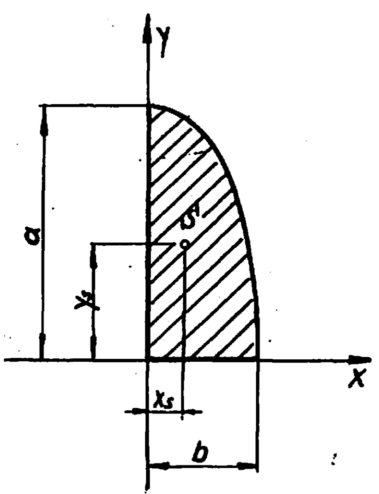
$$y_s = 0$$



11.) Halbe Ellipsenfläche

$$A = \frac{1}{2}\pi ab$$

$$y_s = \frac{4a}{3\pi}$$

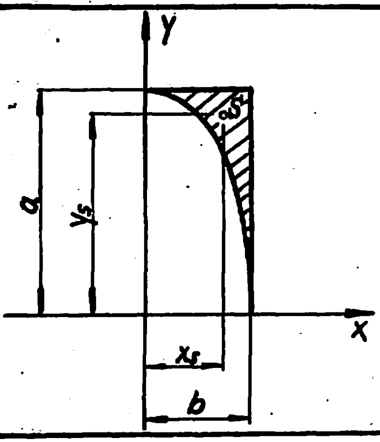


12.) Ein Viertel der Ellipsenfläche

$$A = \frac{1}{4}\pi ab$$

$$x_s = \frac{4b}{3\pi}$$

$$y_s = \frac{4a}{3\pi}$$

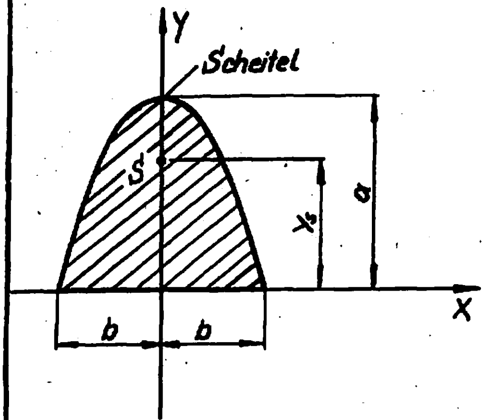


13) Äußerer Teil der Viertelellipsenfläche

$$A = a\cdot b - \frac{1}{4}\pi a b$$

$$x_s = \frac{2}{3}\cdot\frac{b}{4-\pi}$$

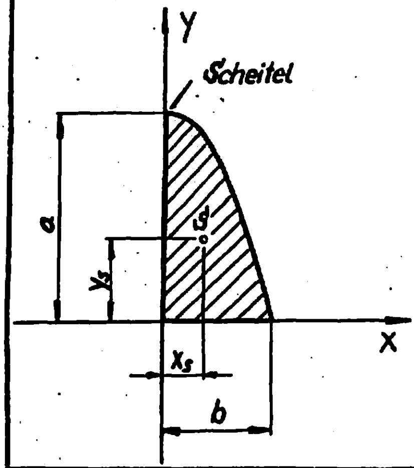
$$y_s = \frac{2}{3}\cdot\frac{a}{4-\pi}$$



14) Parabelfläche

$$A = \frac{4}{3}ab$$

$$y_s = \frac{2}{3}a$$

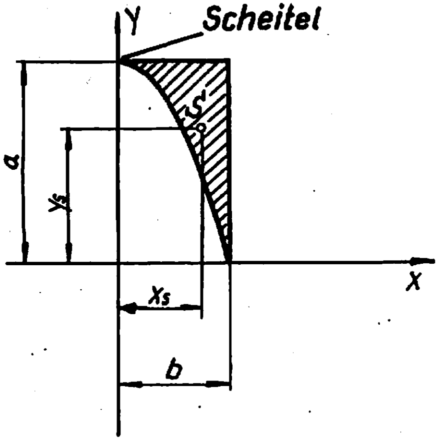


15.) Halbe Parabelfläche

$$A = \frac{2}{3}ab$$

$$x_s = \frac{3}{8}b$$

$$y_s = \frac{2}{5}a$$



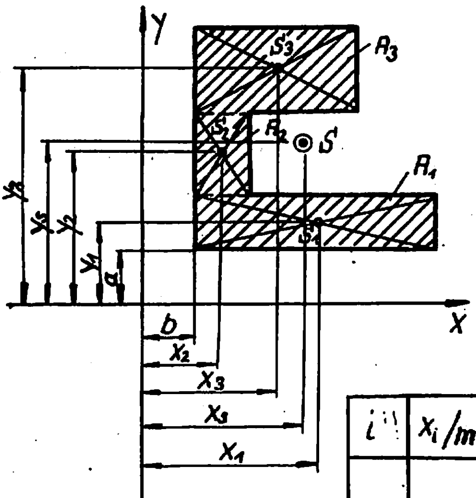
16.) Äußerer Teil der halben Parabelfläche

$$A = a \cdot b - \frac{2}{3} a \cdot b = \frac{1}{3} a \cdot b$$

$$x_s = \frac{3}{4} b$$

$$y_s = \frac{7}{10} a$$

Schwerpunktberechnung zusammengesetzter homogener Flächen



$$x_s = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i}$$

$$y_s = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i}$$

Rechenschema:

i	x_i/mm	y_i/mm	A_i/mm²	$A_i x_i$/mm³	$A_i y_i$/mm³
			$\sum_{i=1}^{n} A_i$	$\sum_{i=1}^{n} A_i x_i$	$\sum_{i=1}^{n} A_i y_i$

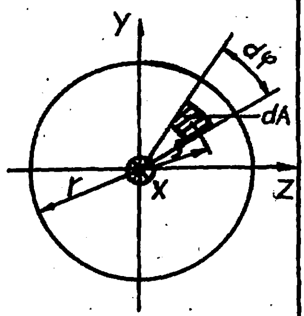
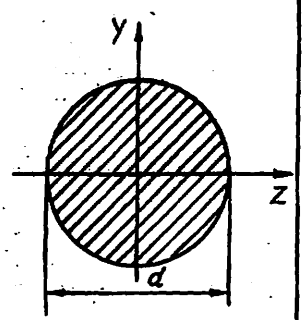
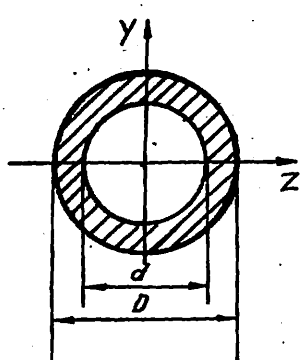
Stahlarten : M Maschinenstahl, E Einsatzstahl,
V Vergütungsstahl, A Automatenstahl
Am Schluß der Bezeichnung angegeben: e einsatzgehärtet
v vergütet

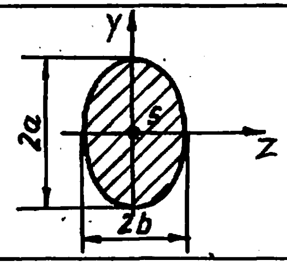
Stahlart	Werkstoffe	Lastfall	Zulässige Spannungen in N mm⁻² Zug und Druck	Biegung	Scherung	Drehung und Schub
		I	110	114	88	60
M	St 34	II	92	95	74	52
A	9 S 20 v	III	50	60	40	33
		I	120	130	96	70
M	St 38	II	96	110	77	60
A	10 S 20 v	III	56	70	45	39
M	St 42	I	125	150	100	80
E	C 10 e	II	109	130	87	70
A	22 S 20 v	III	65	85	50	43
M	St 50	I	155	180	124	95
E	C 15 e	II	135	160	108	83
V	C 22 v	III	75	105	60	54
M	St 60	I	180	210	144	110
A	35 S 20 v	II	157	185	126	95
V	C 35 v	III	86	120	70	62
M	St 70	I	210	245	168	130
A	45 S 20 v	II	175	220	140	113
V	C 45 v	III	100	135	80	73
E	15 Cr 3 e	I	225	250	180	135
A	60 S 20 v	II	170	215	136	117
V	C 60 v	III	100	150	90	79
E	16 MnC 5 e	I	280	330	212	170
V	40 Mn 4 v	II	190	250	150	148
V	34 Cr 4 v	III	140	160	100	85
V	25 Cr Mo 4 v					
E	20 MnCr 5 e	I	310	340	220	160
V	34 Cr Mo 4 v	II	260	280	160	140
		III	150	180	110	78
E	15 Cr Ni 6 e	I	330	360	240	180
V	37 Mn Si 5 v	II	280	300	180	155
V	36 Cr Ni Mo 4 v	III	160	180	120	90
E	18 Cr Ni 8 e	I	350	420	280	215
V	42 Mn V 7 v	II	300	320	180	175
V	41 Cr v	III	190	200	120	90

Stahlart	Werkstoffe	Lastfall	Zulässige Spannungen in Nmm^{-2}			
			Zug und Druck	Biegung	Scherung	Drehung und Schub
E	41 Cr 4e	I	380	500	310	270
V	34 Cr Ni Mo 6v	II	320	350	310	230
V	30 Cr Ni Mo 8v	III	210	230	140	110
GS 38 geglüht		I	110	130	90	66
		II	85	100	70	51
		III	50	60	40	27
GS 45 geglüht		I	130	145	110	77
		I	90	120	80	61
		III	60	70	50	36
Kupfer, hart, gezogen		I	100	100	80	60
		II	45	45	36	27
		III	30	30	24	18
Messing Ms 58 weich		I	100	115	80	67
		II	70	80	56	47
		III	50	58	40	33
AlMg Mn ausgehärtet		I	100	115	80	67
		I	70	80	57	48
		III	50	60	40	34

Festigkeitsart: Werkstoff	Lastfall	Zug in Nmm^{-2}	Druck in Nmm^{-2}	Scher. Schub in Nmm^{-2}	Torsion in ● Nmm^{-2}	○ Nmm^{-2}	■ Nmm^{-2}	I Nmm^{-2}
Gußeisen GG 12 (Für GG 26 und TGW 35 können doppelt so hohe Werte angenommen werden)	I	40	100	40	35	25	45	50
	II	35	80	35	30	22	40	45
	III	25	-	25	20	16	28	30

	Biegung					
	mit Gußhaut			bearbeitet		
Lastfall	●	■	I	●	■	I
I	60	50	40	70	60	50
II	50	45	35	55	50	45
III	35	30	25	40	35	30

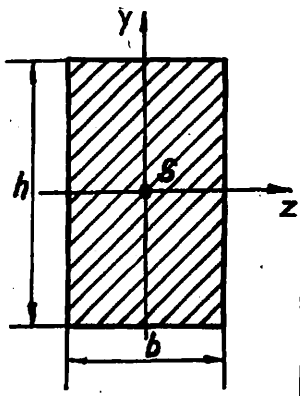
Querschnitt	Torsionsträgheitsmoment J_t/mm^4	Torsionswiderstandsmoment W_t/mm^3
	$J_t = \int_A dA \cdot \rho^2$ gilt für alle Querschnitte	$W_t = \frac{J_t}{r}$ gilt nur für Kreisquerschnitte
	$J_t = \frac{\pi}{32} d^4$	$W_t = \frac{\pi}{16} d^3$
	$J_t = \frac{\pi}{32}(D^4 - d^4)$ Höhlungsverhältnis $\alpha = \frac{d}{D}$ Erfahrungswert : $0{,}4 \leq \alpha \leq 0{,}6$ $J_t = \frac{\pi}{32} D^4 (1-\alpha^4)$	$W_t = \frac{\pi}{16} \cdot \frac{D^4 - d^4}{D}$ $W_t = \frac{\pi}{16} D^3 (1-\alpha^4)$



$$J_t = \frac{\pi}{16} \cdot \frac{a^3 \cdot b^3}{a^2 + b^2}$$

$$W_t = \frac{\pi}{16} a \cdot b^2$$

Für $a \geq b$

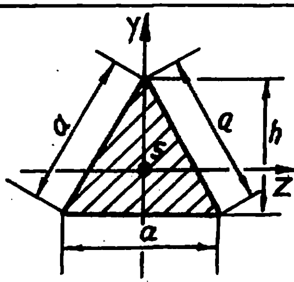


$$J_t = c_1 b^4$$

$$W_t = c_2 b^3$$

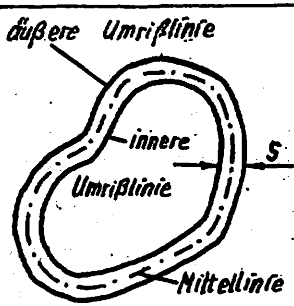
h/b	1	1,5	2	3	4
c_1	0,1406	0,2936	0,457	0,789	1,123
c_2	0,208	0,346	0,493	0,801	1,15

h/b	6	8	10
c_1	1,789	2,456	3,123
c_2	1,789	2,456	3,123



$$J_t = \frac{1}{80}\sqrt{3}\, a^4 = \frac{1}{45}\sqrt{3}\, h^4$$

$$W_t = \frac{1}{20} a^3 = \frac{2}{45}\sqrt{3}\, h^3$$



weitere Flächen siehe Fachliteratur

$$J_t = \frac{2(A_a + A_i) \cdot s \cdot A_m}{U_m}$$

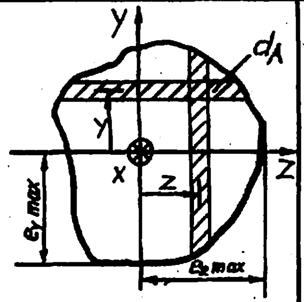
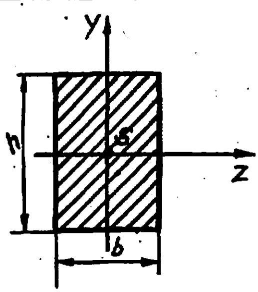
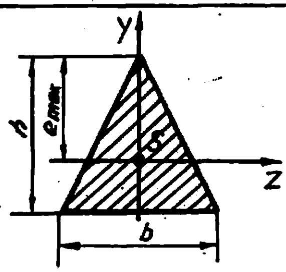
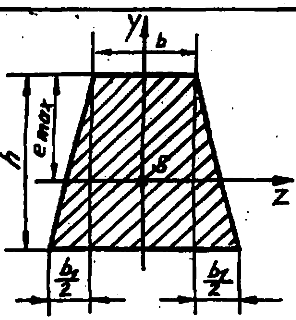
$$W_t = (A_a + A_i) \cdot s$$

A_a ≙ Inhalt der von der äußeren Umrißlinie begrenzten Fläche

A_i ≙ Inhalt der von der inneren Umrißlinie begrenzten Fläche

A_m ≙ Inhalt der von der Mittellinie umgrenzten Fläche $\quad U_m$ ≙ Länge der Mittellinie

Institut für Materialwissenschaft und Werkstofftechnologie Professur für Angewandte Mechanik	Statik und Festigkeitslehre Axiale (äquatoriale) Trägheits- und Widerstandsmomente homogener Flächen - 1

Querschnitt	Axiales Trägheitsmoment J/mm^4	Axiales Widerstandsmoment W/mm^3
	$J_z = \int_A dA \cdot y^2$ $J_y = \int_A dA \cdot z^2$	$W_z = \frac{J_z}{e_{y\,max}}$ $W_y = \frac{J_y}{e_{z\,max}}$
	$J_z = \frac{b \cdot h^3}{12}$ $J_y = \frac{hb^3}{12}$ Für Quadratfläche entsprechend	$W_z = \frac{b \cdot h^2}{6}$ $W_y = \frac{h \cdot b^2}{6}$
	$J_z = \frac{b \cdot h^3}{36}$	$W_z = \frac{b \cdot h^2}{24}$ $e_{max} = \frac{2}{3} h$
	$J_z = \frac{(6 \cdot b^2 + 6 b_1 b + b_1^2) h^3}{36 (2b + b_1)}$	$W_z = \frac{(6b^2 + 6 b_1 b + b_1^2) h^2}{12 (3b + 2b_1)}$ $e_{max} = \frac{(3b + 2b_1) h}{6b + 3b_1}$